Operations/functions defining algebraic/arithmetic expressions

1. The operation Θ is defined by x Θ y = 1/x + 1/y for all nonzero numbers x and y. If z is a number greater than 1, which of the following must be true?

   I. $z \,\theta\, (-z) = 0$

   II. $z \,\theta\, \frac{z}{(z-1)} = 1$

   III. $\frac{2}{z} \,\theta\, \frac{2}{z} = z$

   A. I only
   B. I and II only
   C. I and III only
   D. II and III only
   E. I, II, and III

2. If the operation @ is defined by $x @ y = \sqrt{xy}$ for all positive numbers x and y, then (5@45)@60 =

   (A) 30
   (B) 60
   (C) 90
   (D) $30\sqrt{15}$
   (E) $60\sqrt{15}$

3. In the formula V = 1/(2r)^3, if r is halved, then V is multiplied by

   A. 64
   B. 8
   C. 1
   D. 1/8
   E. 1/64

4. If the operation @ is defined for all integers a and b by a@b=a+b-ab, which of the following statements must be true for all integers a, b and c?

   I. a@b = b@a
   II. a@0 = a
   III. (a@b)@c = a@(b@c)

   (A) I only
   (B) II only
   (C) I and II only
   (D) I and III only
   (E) I, II and III

5. An operation @ is defined by the equation a@b = (a - b) / (a + b), for all numbers a and b such that a ≠ -b. If a ≠ -c and a@c = 0, then c =

   (A) -a
   (B) -1/a
   (C) 0
   (D) 1/a
   (E) a

6. A function f is defined by f(x,y) = x^2 - y. Is the value of f(3,b) less than the value of f(a,3)?

   (1) a < b
   (2) a + 4 < 0

7. If the function Q is defined by the formula Q = 5w/(4x(z^2)), by what factor will Q be multiplied if w is quadrupled, x is doubled, and z is tripled?

   A. 1/9
   B. 2/9
   C. 4/9
   D. 3/9
   E. 2/27

8. The operation x#n for all positive integers greater than 1 is defined in the following manner: x#n = x to the power of x#(n-1)

   If x#1 = x, which of the following expressions has the greatest value?

   A. (3#2)#2
   B. 3#(1#3)
   C. (2#3)#2
   D. 2#(2#3)
   E. (2#2)#3

9. For any numbers a and b, a#b=a + b - ab. If a#b=0, which of the following CANNOT be a value of b?

   A. 2
   B. 1
   C. 0
   D. -1
   E. -3/2

10. For all numbers s and t, the operation * is defined by s*t = (s - 1)(t + 1). If (-2)*x = -12, then x =

    (A) 2
    (B) 3
    (C) 5
    (D) 6
    (E) 11

11. The operation # is defined for all nonzero x and y by x#y = x + x/y. If a>0, then 1#(1#a) =

    A. a
    B. a+1
    C. a/(a+1)
    D. (a+2)/(a+1)
    E. (2a+1)/(a+1)

12. The operation & is defined for all integers a and b by the equation a&b = (a - 1)(b - 1). If x&13 = 96, what is the value of x ?

    (A) 7
    (B) 8
    (C) 9
    (D) 10
    (E) 12

13. If the operation ^ ^ is defined for all x and y by the equation x ^ ^ y = $\frac{x^2y}{2}$, then (2 ^ ^ – 1) ^ ^ (–2 ^ ^1) =

    (A) -4
    (B) -2
    (C) 2
    (D) 4
    (E) 8

14. Operation "#" is defined as adding a randomly selected two digit multiple of 6 to a randomly selected two digit prime number and reducing the result by half. If operation "#" is repeated 10 times, what is the probability that it will yield at least two integers?

A) 0%
B) 10%
C) 20%
D) 30%
E) 40%

15. For any operation ? that acts on two numbers x and y, the commutator is defined as x?y – y?x. For which of the following operations is the commutator not equal to zero for some values of x and y?

I. x?y = xy
II. x?y = $(x - y)^2$
III. x?y = $x^3 + 3x^2y + 3xy^2 + y^3$

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

16. The operation $\otimes$ is defined for all nonzero numbers a and b by $a \otimes b = a/b - b/a$. If x and y are nonzero numbers, which of the following statements must be true?

I. $x \otimes xy = x(1 \otimes y)$
II. $x \otimes y = -(y \otimes x)$
III. $1/x \otimes 1/y = y \otimes x$

A. I only
B. II only
C. III only
D. I and II
E. II and III

17. If operation @X is defined as @X=X+2 if X is even and @X=X−1 if X is odd, what is @(...@(@(@(15)))...) 99 times?

A. 120
B. 180
C. 210
D. 225
E. 250

18. For any integer k greater than 1, the symbol k* denotes the product of all the fractions of the form 1/t, where t is an integer between 1 and k, inclusive. What is the value of 5*/4* ?

A. 5
B. 5/4
C. 4/5
D. 1/4
E. 1/5

19. If $f(x) = x^x$, then f(f(x))=

A. $x^{x^{x^x}}$
B. x^{x^x}
C. $x^{x^{x^2}}$
D. $x^{x^{x+1}}$
E. $(x^x)^x$

20. For all integers x and y, the operation Δ is defined by $x\Delta y = (x+2)^2 + (y+3)^2$. What is the value of integer t?

(1) $t \Delta 2 = 74$
(2) $2 \Delta t = 80$